SEMESTER VI

VI.1 Analysis V

Total marks: 150
Theory: 75
Internal Assessment: 25
Practical: 50
5 Lectures, Practical 2,1 Tutorial (per week per student)

Review of complex plane, sequences and series, connected sets and polygonally connected sets in the complex plane, stereographic projection, analytic polynomials, power series, analytic functions, Cauchy-Riemann equations, functions e^z, $\sin z$ and $\cos z$.

Reference:

[1]: Chapter 1, Chapter 2, Chapter 3.

Line integrals and their properties, closed curve theorem for entire functions, Cauchy integral formula and Taylor expansions for entire functions, Liouville's theorem and the fundamental theorem of algebra.

Reference:

[1]: Chapter 4, Chapter 5.

Power series representation for functions analytic in a disc, analyticity in an arbitrary open set, uniqueness theorem, definitions and examples of conformal mappings, bilinear transformations.

Reference:

[1]: Chapter 6 (Sections 6.1-6.2, 6.3 (up to theorem 6.9), Chapter 9 (Sections 9.2, 9.7-9.8, 9.9 (statement only), 9.10, 9.11 (with examples), 9.13), Chapter 13 (Sections 13.1, 13.2 (up to Theorem 13.11 including examples)).

Fourier series, Piecewise continuous functions, Fourier cosine and sine series, property of Fourier coeffcients, Fourier theorem, discussion of the theorem and its corollary.

Reference: [2].

***Practical / Lab work to be performed on a computer*:**

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

(i) Declaring a complex number e.g. $z_1 = 3 + 4i$, $z_2 = 4 - 7i$. Discussing their algebra $z_1 + z_2$, $z_1 - z_2$, $z_1 \cdot z_2$, and z_1 / z_2 and then plotting them.

(ii) Finding conjugate, modulus and phase angle of an array of complex numbers. e.g., Z = [2+ 3i 4–2i 6+11i 2–5i] (*iii*) Compute the integral over a straight line path between the two specified end points e.g., $\int_C f(z)\, dz$, where C is the straight line path from $a + ib$ to $c + id$.

(iv) Perform contour integration e.g. $\int_C f(z)\, dz$ where C is the Contour given by $g(x, y) = 0$.

(v) Plotting of the complex functions like $f(z) = z$, $f(z) = z^3$, $f(z) = (z^4 - 1)^{1/4}$, etc.

(vi) Finding the residues of the complex function.

(vii) Taylor series expansion of a given function $f(z)$ around a given point z, given the number of terms in the Taylor series expansion. Hence comparing the function and its Taylor series expansion by plotting the magnitude of each. For example

(i) $f(z) = \exp(z)$ around $z = 0$, $n = 40$.

(ii) $f(z) = \exp(z^2)$ around $z = 0$, $n = 160$, etc.

(viii) To perform Laurentz series expansion of a given function $f(z)$ around a given point z, e.g., $f(z) = (\sin z - 1)/z^4$ around $z = 0$, $f(z) = \cot(z)/z^4$ around $z = 0$., etc.

(ix) Computing the Fourier series, Fourier sine series and Fourier cosine series of a function and plotting their graphs.

References:

1. **Joseph Bak and Donald J. Newman,** *Complex analysis* (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.
2. **John P. D'Angelo,** An Introduction to Complex Analysis and Geometry, American Mathematical Society, 2010
3. *Fourier Series*, Lecture notes published by the Institute of Life Long Learning, University of Delhi, Delhi, 2011.

VI.2 Algebra V

Total Marks : 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Properties of external direct products, the group of units modulo n as an external direct product, applications of external direct products to data security, public key cryptography, definition and examples of internal direct products, fundamental theorem of finite abelian groups, definition and examples of group actions, stabilizers and kernels of group actions, permutation representation associated with a given group action.

References:

[1]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1).

[3]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.

Applications of group actions: Cauchy's theorem, Index theorem, Cayley's theorem, conjugacy relation, class equation and consequences, conjugacy in S_n, p-groups, Sylow's theorems and consequences. Definition and examples of simple groups, non-simplicity tests, composition series, Jordan-Holder theorem, solvable groups.

References:

[1]: Chapter 3 (Section 3.4, Exercise 9), Chapter 4 (Sections 4.2-4.3, 4.5-4.6).

[3]: Chapter 25.

Normal operators and self-adjoint operators, unitary and orthogonal operators, matrices of orthogonal and unitary operators, rigid motions, orthogonal operators on R^2, conic sections.

Reference:

[2]: Chapter 6 (Sections 6.4-6.5).

Primary decomposition theorem, theorem on decomposition into sum of diagonalizable and nilpotent operator, cyclic subspaces and annihilators, cyclic decomposition theorem, rational form, invariant factors, Jordan form.

Reference:

[4]: Chapter 6 (Section 6.8), Chapter 7 (Sections 7.1-7.3).

REFERENCES:

1. **David S. Dummit** and **Richard M. Foote**, *Abstract Algebra* (2nd Edition), John Wiley and Sons (Asia) Pte. Ltd, Singapore, 2003.
2. **Stephen H. Friedberg, Arnold J. Insel** and **Lawrence E. Spence**, *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.
3. **Joseph A. Gallian**, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi, 1999.
4. **Kenneth Hoffman** and **Ray Kunze**, *Linear Algebra* (2nd edition), Pearson Education Inc., India, 2005.

Optional Papers VI.3

Optional Paper 1: Discrete Mathematics

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms.

References:

[1]: Chapter 1 (till the end of 1.18), Chapter 2 (Sections 2.1-2.13), Chapter 5 (Sections 5.1-5.11).

[3]: Chapter 1 (Section 1).

Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

References:

[1]: Chapter 6.

[3]: Chapter 1 (Sections 3-4, 6), Chapter 2 (Sections 7-8).

Definition, examples and basic properties of graphs, pseudographs, complete graphs, bi-partite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.

Reference:

[2]: Chapter 9, Chapter 10.

References:

1. **B A. Davey** and **H. A. Priestley**, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1990.
2. **Edgar G. Goodaire** and **Michael M. Parmenter**, *Discrete Mathematics with Graph Theory* (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Reprint 2003.
3. **Rudolf Lidl** and **Günter Pilz**, *Applied Abstract Algebra* (2nd Edition), Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

Optional Paper 2 : Mathematical Finance

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Basic principles: Comparison, arbitrage and risk aversion, Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR. Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, immunization, convexity, putable and callable bonds.

References:

[1]: Chapter 1, Chapter 2, Chapter 3, Chapter 4.

Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharpe index. Capital Asset Pricing Model (CAPM), betas of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index.

References:

[1]: Chapter 6, Chapter 7, Chapter 8 (Sections 8.5--8.8).

[3]: Chapter 1 (for a quick review/description of expectation etc.)

Forwards and futures, marking to market, value of a forward/futures contract, replicating portfolios, futures on assets with known income or dividend yield, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, interest rate futures, swaps. Lognormal distribution, Lognormal model / Geometric Brownian Motion for stock prices, Binomial Tree model for stock prices, parameter estimation, comparison of the models. Options, Types of options: put / call, European / American, pay off of an option, factors affecting option prices, put call parity.

References:

[1]: Chapter 10 (except 10.11, 10.12), Chapter 11 (except 11.2 and 11.8)

[2]: Chapter 3, Chapter 5, Chapter 6, Chapter 7 (except 7.10 and 7.11), Chapter 8, Chapter 9

[3]: Chapter 3

REFERENCES:

1. **David G. Luenberger**, *Investment Science*, Oxford University Press, Delhi, 1998.
2. **John C. Hull**, *Options, Futures and Other Derivatives* (6th Edition), Prentice-Hall India, Indian reprint, 2006.
3. **Sheldon Ross**, *An Elementary Introduction to Mathematical Finance* (2nd Edition), Cambridge University Press, USA, 2003.

Optional Paper 3 : Mechanics

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Moment of a force about a point and an axis, couple and couple moment, Moment of a couple about a line, resultant of a force system, distributed force system, free body diagram, free body involving interior sections, general equations of equilibrium, two point equivalent loading, problems arising from structures, static indeterminacy.

Reference:

[1]: Chapter 3, Chapter 4, Chapter 5.

Laws of Coulomb friction, application to simple and complex surface contact friction problems, transmission of power through belts, screw jack, wedge, first moment of an area and the centroid, other centers, Theorem of Pappus-Guldinus, second moments and the product of area of a plane area, transfer theorems, relation between second moments and products of area, polar moment of area, principal axes.

Reference:

[1]: Chapter 6 (Sections 6.1-6.7), Chapter 7

Conservative force field, conservation for mechanical energy, work energy equation, kinetic energy and work kinetic energy expression based on center of mass, moment of momentum equation for a single particle and a system of particles, translation and rotation of rigid bodies, Chasles' theorem, general relationship between time derivatives of a vector for different references, relationship between velocities of a particle for different references, acceleration of particle for different references.

Reference:

[1]: Chapter 11, Chapter 12 (Sections 12.5-12.6), Chapter 13.

REFERENCES:

1. **I.H. Shames** and **G. Krishna Mohan Rao**, *Engineering Mechanics: Statics and Dynamics* (4th Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2009.
2. **R.C. Hibbeler** and **Ashok Gupta**, *Engineering Mechanics: Statics and Dynamics* (11th Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi.

Optional Paper 4 : Number Theory

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Linear Diophantine equation, prime counting function, statement of prime number theorem, Goldbach conjecture, linear congruences, complete set of residues, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

References:

[1]: Chapter 2 (Section 2.5), Chapters 3 (Section 3.3), Chapter 4 (Sections 4.2 and 4.4), Chapter 5 (Section 5.2 excluding pseudoprimes, Section 5.3).

[2]: Chapter 3 (Section 3.2).

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Möbius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced set of residues, some properties of Euler's phi-function.

References:

[1]: Chapter 6 (Sections 6.1-6.3), Chapter 7.

[2]: Chapter 5 (Section 5.2 (Definition 5.5-Theorem 5.40), Section 5.3 (Theorem 5.15-Theorem 5.17, Theorem 5.19)).

Order of an integer modulo *n*, primitive roots for primes, composite numbers having primitive roots, Euler's criterion, the Legendre symbol and its properties, quadratic reciprocity, quadratic congruences with composite moduli. Public key encryption, RSA encryption and decryption, the equation $x^2 + y^2 = z^2$, Fermat's Last Theorem.

Reference:

[1]: Chapters 8 (Sections 8.1-8.3), Chapter 9, Chapter 10 (Section 10.1), Chapter 12.

REFERENCES:

1. **David M. Burton**, *Elementary Number Theory* (6th Edition), Tata McGraw-Hill Edition, Indian reprint, 2007.
2. **Neville Robinns**, *Beginning Number Theory* (2nd Edition), Narosa Publishing House Pvt. Limited, Delhi, 2007.

VI.4 Credit course III.2