



**Fourth Semester B.E. Degree Examination, June/July 2015**  
**Engineering Mathematics - IV**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.****PART - A**

- 1 a. Obtain  $y(0.2)$  using Picards method upto second iteration for the initial value problem  
 $\frac{dy}{dx} = x^2 - 2y$   $y(0) = 1$ . (06 Marks)
- b. Solve by Eulers modified method to obtain  $y(1.2)$  given  $y' = \frac{y^2 x}{y-x}$   $y(1) = 2$ . (07 Marks)
- c. Using Adam Bash forth method obtain  $y$  at  $x = 0.8$  given  $\frac{dy}{dx} = x - y^2$ ,  $y(0) = 0$ ,  $y(0.2) = 0.02$ ,  $y(0.4) = 0.0795$  and  $y(0.6) = 0.1762$ . (07 Marks)
- 2 a. Solve by 4<sup>th</sup> order Runge Kutta method simultaneous equations given by  
 $\frac{dx}{dt} = y - 1$ ,  $\frac{dy}{dt} = x + 1$  with  $x = 1 = y$  at  $t = 0$ , obtain  $y(0.1)$  and  $x(0.1)$ . (06 Marks)
- b. Solve  $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ ,  $y(0) = 1$ ,  $y'(0) = 0$ . Evaluate  $y(0.2)$  correct to four decimal places, using Runge Kutta method of fourth order. (07 Marks)
- c. Solve for  $x = 0.4$  using Milnes predictor corrector formula for the differential equation  $y'' + xy' + y = 0$  with  $y(0) = 1$ ,  $y(0.1) = 0.995$ ,  $y(0.2) = 0.9802$  and  $y(0.3) = 0.956$ . Also  $z(0) = 0$ ,  $z(0.1) = -0.0995$ ,  $z(0.2) = -0.196$ ,  $z(0.3) = -0.2863$ . (07 Marks)
- 3 a. Verify whether  $f(z) = \sin 2z$  is analytic, hence obtain the derivative. (06 Marks)
- b. Determine the analytic function  $f(z)$  whose imaginary part is  $\frac{y}{x^2 + y^2}$ . (07 Marks)
- c. Define a harmonic function. Prove that real and imaginary parts of an analytic function are harmonic. (07 Marks)
- 4 a. Under the mapping  $w = e^z$ , find the image of i)  $1 \leq x \leq 2$  ii)  $\frac{\pi}{3} < y < \frac{\pi}{2}$ . (06 Marks)
- b. Find the bilinear transformation which maps the points 1, i, -1 from  $z$  plane to 2, i, -2 into  $w$  plane. Also find the fixed points. (07 Marks)
- c. State and prove Cauchy's integral formula. (07 Marks)

**PART - B**

- 5 a. Prove  $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$ . (06 Marks)
- b. Prove  $(n+1) P_n(x) = (2n+1) x P_n(x) - n P_{n-1}(x)$ . (07 Marks)
- c. Explain the following in terms of Legendres polynomials.  
 $x^4 + 3x^3 - x^2 + 5x - 2$  (07 Marks)

- 6 a. A class has 10 boys and 6 girls. Three students are selected at random one after another. Find the probability that i) first and third are boys, second a girl ii) first and second are of same sex and third is of opposite sex. (06 Marks)
- b. If  $P(A) = 0.4$ ,  $P(B/A) = 0.9$ ,  $P(\bar{B}/\bar{A}) = 0.6$ . Find  $P(A/B)$ ,  $P(A/\bar{B})$ . (07 Marks)
- c. In a bolt factory machines A, B and C manufacture 20%, 35% and 45% of the total of their outputs 5%, 4% and 2% are defective. A bolt is drawn at random found to be defective. What is the probability that it is from machine B? (07 Marks)

- 7 a. A random variable  $x$  has the following distribution :

|          |     |     |     |     |      |     |     |
|----------|-----|-----|-----|-----|------|-----|-----|
| $x :$    | -2  | -1  | 0   | 1   | 2    | 3   | 4   |
| $P(x) :$ | 0.1 | 0.1 | $k$ | 0.1 | $2k$ | $k$ | $k$ |

Find  $k$ , mean and S.D of the distribution.

(06 Marks)

- b. The probability that a bomb dropped hits the target is 0.2. Find the probability that out of 6 bombs dropped i) exactly 2 will hit the target ii) atleast 3 will hit the target. (07 Marks)
- c. Find the mean and variance of the exponential distribution. (07 Marks)

- 8 a. A die is tossed 960 times and 5 appear 184 times. Is the die biased? (06 Marks)
- b. Nine items have values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from assumed of mean of 47.5. ( $\gamma = 8$ ,  $t_{0.05} = 2.31$ ). (07 Marks)
- c. A set of 5 similar coins tossed 320 times gives following table.

|              |   |    |    |     |    |    |
|--------------|---|----|----|-----|----|----|
| No. of heads | 0 | 1  | 2  | 3   | 4  | 5  |
| Freq.        | 6 | 27 | 72 | 112 | 71 | 32 |

Test the hypothesis that data follows binomial distribution (Given  $\gamma = 5$ ,  $\chi_{0.05}^2 = 11.07$ )

(07 Marks)

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