USN

10MAT31

Third Semester B.E. Degree Examination, June/July 2015

Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Expand $f(x) = x \sin x$ as a Fourier series in the interval $(-\pi, \pi)$, Hence deduce the following:

i) $\frac{\pi}{2} = 1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7}$

ii) $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - + \ \ldots$ **(07 Marks)**

b. Find the half–range Fourier cosine series for the function

$$f(x) = \begin{cases} kx, & 0 \le x \le \ell/2 \\ k(\ell - x), & \ell/2 < x \le \ell \end{cases}$$

Where k is a non-integer positive constant. **(06 Marks)**

c. Find the constant term and the first two harmonics in the Fourier series for f(x) given by the following table.

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
F(x) :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(07 Marks)

2 a. Find the Fourier transform of the function $f(x) = xe^{-a|x|}$ **(07 Marks)**

b. Find the Fourier sine transforms of the

Functions $f(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x \ge a \end{cases}$ **(06 Marks)**

c. Find the inverse Fourier sine Transform of

$F_s(\alpha) = \frac{1}{\alpha} e^{-a\alpha} \quad a > 0.$ **(07 Marks)**

3 a. Find various possible solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ by separable variable method. **(07 Marks)**

b. Obtain solution of heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial t^2}$ subject to condition $u(0,t) = 0$, $u(\ell, t) = 0$, $u(x, 0) = f(x)$. **(06 Marks)**

c. Solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to condition $u(0, y) = u(\ell, y) = 0$, $u(x, 0) = 0$, $u(x, a) = \sin\left(\frac{\pi x}{\ell}\right)$. **(07 Marks)**

4 a. The pressure P and volume V of a gas are related by the equation $PV^r = K$, where r and K are constants. Fit this equation to the following set of observations (in appropriate units)

P:	0.5	1.0	1.5	2.0	2.5	3.0
V:	1.62	1.00	0.75	0.62	0.52	0.46

(07 Marks)

b. Solve the following LPP by using the Graphical method :
Maximize : $Z = 3x_1 + 4x_2$
Under the constraints $4x_1 + 2x_2 \le 80$
$2x_1 + 5x_2 \le 180$
$x_1, x_2 \ge 0$. (06 Marks)

c. Solve the following using simplex method
Maximize : $Z = 2x + 4y$, subject to the
Constraint : $3x + y \le 2z$, $2x + 3y \le 24$, $x \ge 0$, $y \ge 0$. (07 Marks)

PART – B

5 a. Using the Regular – Falsi method, find a real root (correct to three decimal places) of the equation $\cos x = 3x - 1$ that lies between 0.5 and 1 (Here, x is in radians). (07 Marks)

b. By relaxation method
Solve : $-x + 6y + 27z = 85$, $54x + y + z = 110$, $2x + 15y + 6z = 72$. (06 Marks)

c. Using the power method, find the largest eigen value and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
taking $[1, 1, 1]^T$ as the initial eigen vectors. Perform 5 iterations. (07 Marks)

6 a. From the data given in the following Table ; find the number of students who obtained
(i) Less than 45 marks ii) between 40 and 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 - 80
No. of Students	31	42	51	35	31

(07 Marks)

b. Using the Lagrange's formula, find the interpolating polynomial that approximates to the function described by the following table:

x	0	1	2	3	4
f(x)	3	6	11	18	27

Hence find $f(0.5)$ and $f(3.1)$. (06 Marks)

c. Evaluate $\int_0^1 \frac{x}{1+x^2}\,dx$ by using Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule, dividing the interval into 3 equal parts.
Hence find an approximate value of $\log\sqrt{2}$. (07 Marks)

7 a. Solve the one – dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$
Subject to the boundary conditions $u(0, t) = 0$, $u(1, t) = 0$, $t \ge 0$ and the initial conditions $u(x, 0) = \sin \pi x$, $\frac{\partial u}{\partial t}(x, 0) = 0$, $0 < x < 1$. (07 Marks)

b. Consider the heat equation $2\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ under the following conditions:

i) $u(0, t) = u(4, t) = 0, \ t \geq 0$

ii) $u(x, 0) = x(4 - x), \ 0 < x < 4$.

Employ the Bendre – Schmidt method with $h = 1$ to find the solution of the equation for $0 < t \leq 1$. **(06 Marks)**

c. Solve the two – dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior pivotal points of the square region shown in the following figure. The values of u at the pivotal points on the boundary are also shown in the figure. **(07 Marks)**

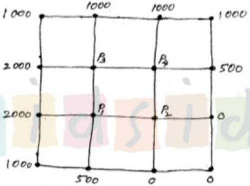

Fig. Q7 (c)

8 a. State and prove the recurrence relation of Z – Transformation hence find $Z_T(n^p)$ and $Z_T\left[\cosh\left(\frac{n\pi}{2} + \theta\right)\right]$. **(07 Marks)**

b. Find $Z_T^{-1}\left[\frac{z^3 - 20z}{(z-2)^3 (z-4)}\right]$ **(06 Marks)**

c. Solve the difference equation

$y_{n+2} - 2y_{n+1} - 3y_n = 3^n + 2n$

Given $y_0 = y_1 = 0$. **(07 Marks)**

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