

Seat No.	
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[4757]-1071

S.E. (Computer Engineering/Information Technology)
(Second Semester) EXAMINATION, 2015
ENGINEERING MATHEMATICS-III
(2012 PATTERN)

Time : Two Hours**Maximum Marks : 50**

- N.B. :-** (i) Attempt *four* questions : Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
(ii) Neat diagrams must be drawn wherever necessary.
(iii) Figures to the right indicate full marks.
(iv) Use of electronic non-programmable calculator is allowed.
(v) Assume suitable data if necessary.

1. (a) Solve (any two) : [8]

(i) $(D^2 + 9)y = x^3 - \cos 3x$

(ii) $(D^2 + 2D + 1)y = e^{-x} \log x$

(iii) $(2x + 1)^2 \frac{d^2 y}{dx^2} - 6(2x + 1) \frac{dy}{dx} + 16y = 8(2x + 1)^2$.

(b) Find Fourier sine transform of $f(x) = e^{-x} \cos x$, $x > 0$. [4]

Or

2. (a) A resistance of 50 ohms, an inductor of 2 henries and a 0.005 farad capacitor are connected in series with e.m.f. of 40 volts and an open switch. Find the instantaneous charge and current after the switch is closed at $t = 0$, assuming that at that time charge on capacitor is 4 coulomb. [4]

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(b) Solve (any one) : [4]

(i) Find z -transform of $f(k) = \frac{\sin ak}{k}$, $k > 0$.

(ii) Find inverse z -transform of $\frac{3z^2 + 2z}{z^2 + 3z + 2}$, $1 < |z| < 2$.

(c) Solve difference equation : [4]

$$f(k + 2) - 3f(k + 1) + 2f(k) = 0, f(0) = 0, f(1) = 1.$$

3. (a) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

(b) A manufacturer of electronic goods has 4% of his product defective. He sells the articles in packets of 300 and guarantees 90% good quality. Determine the probability that a particular packet will violate the guarantee. [4]

(c) Find the directional derivative of $xy^2 + yz^3$ at $(2, -1, 1)$ along the line $2(x - 2) = (y + 1) = (z - 1)$. [4]

Or

4. (a) In an intelligence test administered to 1000 students the average score was 42 and standard deviation 24. Find the number of students with score lying between 30 and 54.

(Given : For $z = 0.5$, area = 0.1915). [4]

(b) Prove (any one) : [4]

(i) $\nabla^2 \left(\frac{\bar{a} \cdot \bar{b}}{r} \right) = 0$

(ii) $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r} \right) = \frac{\bar{a}}{r} + \frac{(\bar{a} \cdot \bar{r})\bar{r}}{r^3}.$

- (c) Show that $\vec{F} = r^2 \vec{r}$ is conservative. Obtain the scalar potential associated with it. [4]

5. (a) Evaluate : [4]

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$ and

C is the parabolic arc $y = x^2$ joining (0, 0) and (1, 1).

- (b) Using Stokes theorem, evaluate : [5]

$$\int_C (x + y)dx + (2x - z)dy + (y + z)dz$$

where C is the curve given by

$$x^2 + y^2 + z^2 - 2ax - 2ay = 0, \quad x + y = 2a.$$

- (c) Use divergence theorem to evaluate :

$$\iiint_S (x\vec{i} - 2y^2\vec{j} + z^2\vec{k}) \cdot d\vec{s}$$

where s is the surface bounded by the region $x^2 + y^2 = 1$ and $z = 0$ and $z = 1$. [4]

Or

6. (a) Apply Green's theorem to evaluate :

$$\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$$

where C is the boundary of the area enclosed by the x-axis and the upper-half of the circle $x^2 + y^2 = 16$. [4]

- (b) Using Stokes theorem, evaluate

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{s}$$

where $\vec{F} = 3y\vec{i} - xz\vec{j} + yz^2\vec{k}$ and 's' is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$. [5]

- (c) Show that : [4]

$$\iiint_V \frac{2}{r} dv = \iint_S \frac{\vec{r} \cdot \hat{n}}{r} ds.$$

7. (a) Find the imaginary part of the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2$. [4]
 (b) Evaluate :

$$\oint_C \frac{z^2 + 1}{z^2 - 1} dz$$

where C is the circle : [4]

$$|z - 1| = 1.$$

- (c) Find the bilinear transformation, which maps the points $z = -1, 0, 1$ on to the points $w = 0, i, 3i$ respectively. [5]

Or

8. (a) Show that analytic function $f(z)$ with constant amplitude is constant. [4]
 (b) Evaluate the following integral using residue theorem :

$$\oint_C \frac{4 - 3z}{z(z - 1)(z - 2)} dz$$

where C is the circle :

$$|z| = \frac{3}{2}. \quad [4]$$

- (c) Find the image of the straight line $y = 3x$ under the transformation

$$w = \frac{z - 1}{z + 1}. \quad [5]$$