FE Sem-II CB 17/5/17

Sub: Applied Mathematics-II

Q.P. Code :16281

Max. Marks: 80 **Time: 3 hrs.**

N.B. : 1. Q1 is compulsory
2. Attempt any three questions from Q2 to Q6.

Q1. a) Show that $\int_0^\infty 3^{-4x^2}\,dx = \frac{\sqrt{\pi}}{4\sqrt{\log 3}}$ 3

b) Solve $(2y^2 - 4x + 5)dx = (y - 2y^2 - 4xy)dy$ 3

c) Solve the ODE $(D-1)^2(D^2+1)^2 y = 0$ 3

d) Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\,dy\,dx$ 3

e) Evaluate $\int_0^1 \frac{x^a - 1}{\log x}dx$ 4

f) Find the length of the cycloid from one cusp to the next, where $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ 4

Q2. a) Solve $(D^2 - 3D + 2)y = 2e^x \sin\left(\frac{x}{2}\right)$ 6

b) Using DUIS prove that $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\,dx = \frac{\sqrt{\pi}}{2}e^{-2a}$, $a > 0$ 6

c) Change the order of integration and evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$ 8

Q3. a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{(x+y+z+1)^3}\,dz\,dy\,dx$ 6

b) Find the mass of the leminiscate $r^2 = a^2\cos 2\theta$ if the density at any point is proportional to the square of the distance from the pole. 6

c) Solve $x^2\frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{y}{x} = 4\log x$ 8

Q4. a) Prove that for an astroid $x^{2/3} + y^{2/3} = a^{2/3}$, the line $\theta = \frac{\pi}{6}$ divide the arc in the first quadrant in a ratio 1:3. 6

b) Solve $(D^2 - 7D - 6)y = (1 + x^2)e^{2x}$ 6

c) Apply Runge Kutta method of fourth order to find an approximate value of y when $x = 0.4$ given that $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y = 1$ when $x = 0$ taking $x = 0.2$ 8

Q5. a) Use Taylor's series method to find a solution of $\frac{dy}{dx} = 1 + xy$, $y(0) = 0$ for $x = 0.2$ taking $h = 0.1$ correct to four decimal places. 6

b) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \frac{1}{1 + \sin x}$ 6

c) Compute the value of $\int_0^{1.4} \left(\sin x - \ln x + e^x\right) dx$ using (i) Trapezoidal rule (ii) Simpson's $(1/3)^{rd}$ rule (iii) Simpson's $(3/8)^{th}$ rule by dividing into six subintervals. 8

Q6. a) Using Beta function evaluate $\int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^2 6\theta \, d\theta$

b) Evaluate $\int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{y}^{\sqrt{a^2 - y^2}} \log\left(x^2 + y^2\right) dx \, dy$ by changing to polar coordinates 6

c) Evaluate $\iiint_V x^2 y z \, dx \, dy \, dz$ over the volume bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ 8