

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-III EXAMINATION - WINTER 2015

Subject Code:130001

Date:31/12/2015

Subject Name: Mathematics-III

Time: 2:30pm to 5:30pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- 1 Solve $\frac{dy}{dx} = \frac{e^x}{e^y} + x^2 e^{-y}$. 14
 - 2 Find $L(e^{-3t} t^{3/2})$
 - 3 Find $L^{-1}(\frac{6(s+1)}{s^4})$
 - 4 Find value of $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$.
 - 5 Determine singular points of differential equation $x(x-1)^3 y' + 2(x-1)^3 y' + 3y = 0$.
 - 6 Solve $(D^4 - 16)y = 0$.
 - 7 Using Bessel's function of the first kind prove that $J_0(0) = 1$.
- Q.2**
- (1) Find $L(\frac{e^{-bt}-e^{-at}}{t})$. 02
 - (2) Find the Laplace transform of the periodic function $f(t) = 3t, 0 < t < 2$
 $= 6, 2 < t < 4$. 05
 - (b) (1) Solve $(D^2 - 5D + 6)y = e^{2x} \sin 2x$. 04
 - (2) Solve $y dx - x dy + \log x dx = 0$. 03
- OR
- (b) (1) Find the power series solution about $x=0$ of $y'' + xy' + x^2 y = 0$. 04
 - (2) Find $L^{-1}(\frac{4s+5}{(s-1)^2 (s+2)})$ 03
- Q.3**
- (1) Solve $x^2 \frac{d^2 y}{dx^2} + 6x \frac{dy}{dx} + 6y = x \log x$. 04
 - (2) Solve $\frac{y^2}{x} dx + (1 + 2y \log x) dy = 0$ 03
 - (b) (1) Using method of undetermined co-efficients solve the differential equation $y'' + 9y = 7e^{3x} + 3x^2 + 4, y(0) = 0, y'(0) = 1$. 04
 - (2) Find Orthogonal trajectories of $r^n = a^n \cos n\theta$. 03

OR

Q.3 (a) (1) Using Laplace transform solve **04**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 0, x'(0) = 1$$

(2) Using convolution theorem find $L^{-1}\left(\frac{2}{(s^2+1)(s^2+4)}\right)$. **03**

(b) (1) Express the following in terms of unit step function **04**

$$f(t) = t - 1, 1 < t < 2 \\ = 3 - t, 2 < t < 3$$

(2) Find $L(te^{-t} \cos ht)$ **03**

Q.4 (a) Find the general solution of $2x^2y'' + xy' + (x^2 - 1)y = 0$ by using Frobenius method. **07**

(b) (1) Show that $x^4 = \frac{1}{35} [8P_4(x) + 20P_2(x) + 7P_0(x)]$ **04**

(2) Show that $\int_{-1}^1 P_m(x) \cdot P_n(x) dx = 0$ if $m \neq n$. **03**

OR

Q.4 (a) Find series solution of the differential equation $y'' + xy = 0$ **07**

(b) (1) State Legendre duplication formula and hence prove that **04**

$$\beta(m, m) \cdot \beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \pi m^{-1} \cdot 2^{1-4m}$$

(2) Evaluate $\int_3^7 \sqrt{(x-3)(7-x)} dx$ **03**

Q.5 (a) Expand $f(x)$ as a fourier series in the interval $(0, 2\pi)$ if **07**

$$f(x) = -\pi, 0 < x < \pi \\ = x - \pi, \pi < x < 2\pi \text{ hence show that}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$$

(b) Express the function $f(x) = -e^{kx}, x < 0$ **07**

$$= e^{-kx}, x > 0 \text{ as fourier integral and}$$

$$\text{hence show that } \int_0^{\infty} \frac{w \sin wx}{w^2 + k^2} dw = \frac{\pi}{2} \cdot e^{-kx} \text{ if } x > 0, k > 0.$$

OR

Q.5 (a) Solve by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ **07**

(b) Find half range cosine series for the function $f(x) = (x-1)^2 \sin 0 < x < 1$ and deduce that **07**

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$
