Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – SUMMER • 2014

Subject Code: 130001 **Date: 02-06-2014**

Subject Name: Mathematics-III

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find the general solution of the differential equation $y' = e^{2x+3y}$. **02**

(ii) Find the particular solution of the differential equation $y''+4y = 2\sin 3x$ by using method of undetermined coefficients. **02**

(iii) Find the inverse Laplace transform of following function: **03**

$$\frac{s}{s^2-3s+2}$$

(b) (i) Define Ordinary Point of the differential equation $y''+P(x)y'+Q(x)y = 0$. **02**

(ii) Find the value of $\left|\frac{3}{4}\right|\frac{1}{4}$. **02**

(iii) Express the function $f(x) = x$ as a Fourier series in interval $[-\pi, \pi]$. **03**

Q.2 **(a)** (i) Evaluate: $\int_0^\infty x^2 e^{-x^4} dx$. **02**

(ii) Solve: $\left(D^4-1\right)y = 0$. **02**

(iii) Solve the partial differential equation: $u_{xy} = x^3 + y^3$. **03**

(b) (i) Find the Laplace transforms of function $f(t) = t^5 + \cos 5t + e^{-100t}$ **03**

(ii) Using method of variation of parameters solve the differential equation $y''+4y = \tan 2x$. **04**

OR

(b) (i) Find the Laplace transforms of the function $f(t) = t\cosh t$. **03**

(ii) Solve: $\left(D^2-25\right)y = \cos 5x$ **04**

Q.3 **(a)** Find the Laplace transforms of following functions: **07**

(i) $\cos^3 t$ (ii) $\sin^2 t$.

(b) State Convolution Theorem and using it find inverse Laplace transform of function $f(t) = \dfrac{s^2}{\left(s^2+4\right)\left(s^2+9\right)}$ **07**

OR

Q.3 **(a)** Using Laplace transform solve the differential equation: **07**

$y''+5y'+6y = e^{-t}$, $y(0) = 0$, $y'(0) = -1$.

	(b)	Evaluate: (i) $\int_{-1}^{1}\left(1-x^2\right)^n dx$,where n is a positive integer. (ii) $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\,d\theta\times\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{\sin\theta}}d\theta.$	**07**
Q.4	**(a)**	(i) Prove that $J_{\frac{3}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x}-\cos x\right]$	**03**
		(ii) $P_n\left(-1\right)^n=\left(-1\right)^n$.	**04**
	(b)	(i) Solve the differential equation $y''+xy=0$ by the power series method.	**03**
		(ii) State Rodrigue's Formula and using it compute $P_0\left(x\right),P_1\left(x\right)$.	**04**

OR

Q.4	**(a)**	(i) Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}$.	**03**
		(ii) Solve : $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$.	**04**
	(b)	(i) If $y_1=x$ is one solution of $x^2y''+xy'-y=0$ then find the second solution.	**03**
		(ii) Solve : $\left(2x+5\right)^2\frac{d^2y}{dx^2}-6\left(2x+5\right)\frac{dy}{dx}+8y=6x$.	**04**
Q.5	**(a)**	(i) Find half range cosine series for the function $f(x)=e^x$ in interval $\left[0,2\right]$.	**03**
		(ii) Express the function $f(x)=x-x^2$ as a Fourier series in interval $\left[-\pi,\pi\right]$.	**04**
	(b)	(i) Evaluate: $\int_0^1\left(x\log x\right)^3dx.$	**03**
		(ii) By using the relation between Beta and Gamma function prove that $\beta\left(m,n\right)\beta\left(m+n,p\right)\beta\left(m+n+p,q\right)=\frac{\Gamma(m)\Gamma(n)\Gamma(p)\Gamma(q)}{\Gamma(m+n+p+q)}.$	**04**

OR

Q.5	**(a)**	Solve completely the equation $\frac{\partial^2y}{\partial t^2}=c^2\frac{\partial^2y}{\partial x^2}$ representing the vibrations of a string of length l fixed at both ends given that, $y\left(0,t\right)=y\left(l,t\right)=0,y\left(x,0\right)=f\left(x\right),\frac{\partial y}{\partial t}\left(x,0\right)=0,0<x<l.$	**07**
	(b)	Find the Fourier Transform of the function f defined as follows: $f\left(x\right)=\begin{matrix} x & \lvert x\rvert<a; \\ 0 & \lvert x\rvert>a. \end{matrix}$	**07**

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